

Supersymmetric Tuned Inflation

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Abstract

We address an issue what kind of tuning realizes a flat potential for inflaton in supergravity. We restrict ourselves to the case that inflationary dynamics is not affected by separate supersymmetry-breaking effects, and consider examples of a single chiral superfield as an inflaton with its vacuum of the vanishing cosmological constant. The examples potentially include the cases of Polonyi field or right-handed sneutrino as an inflaton.

1 Introduction

Slow-roll inflation [1] takes place when the potential is flat enough, which is realized through tuning the form of potential by symmetry or by hand.¹ In non-supersymmetric or supersymmetry-broken² cases, the required tuning is rather straightforward to flatten a potential with the kinetic term intact. On the other hand, in the supersymmetry-invariant case, the Kähler potential is to be tuned, which affects not only the potential but also the kinetic function in supergravity. This complicates the necessary tuning to induce supergravity inflation.

In this paper, we address an issue what kind of tuning realizes a flat potential for inflation in supergravity. We restrict ourselves to the case of supersymmetry intact. Namely, we assume that inflationary dynamics is not affected by separate supersymmetry-breaking effects if any.

The supersymmetric inflation models with chiral superfields may be characterized by the number of superfields and the lowest monomial degree of their superpotential around the vacuum. We examine examples of a single chiral superfield³ as an inflaton, for simplicity, with its vacuum of the vanishing cosmological constant. The examples potentially include the cases of Polonyi field or right-handed sneutrino as an inflaton.

The rest of the paper goes as follows: In section 2, we recapitulate the form of scalar potential in supergravity. Then we deal with two cases of inflation with large and small field variations in turn. In section 3, a tuning for large-field inflation in effective field theory is applied to the case of supergravity and quadratic potentials for chaotic inflation are obtained.⁴ The inflaton turns out to be a Polonyi field⁵ in one class of models and a ‘right-handed sneutrino’⁶ in another, though we do not try to construct realistic models

¹ Fine-tuning problem in inflationary models seems subtle. Although required parameter tuning itself tends to be unnatural in field theory landscape, the tuning is possibly advantageous to realize (infinitely) large volume of (our) universe, which may be environmentally favorable and compensate unnaturalness of the delicate parameter choices. For a positive use of such a tuning relevant to particle physics, see Ref.[2].

² See Ref.[3] for use of separate supersymmetry-breaking effects (besides inflaton (temporary) supersymmetry breaking) to induce inflation.

³See Ref.[4] for an example of multi-field case.

⁴ See Ref.[5, 6] for some realizations of chaotic inflation in supergravity.

⁵ See Ref.[7] for supersymmetry-breaking inflation.

⁶ See Ref.[6] for sneutrino inflation.

corresponding to those fields in this paper. Section 4 provides ‘saddle point’ [3] models as examples of small-field inflation with a wider range of physical parameter scales compared to the above large-field case. Section 5 concludes the paper.

2 The supergravity potential

Let us adopt a chiral superfield ϕ with a superpotential $W(\phi)$ and a Kähler potential

$$K = -3 \ln \left(-\frac{\Omega}{3} \right), \quad (1)$$

where $\Omega = -3 + \dots$ denotes a real function of ϕ and the reduced Planck unit $M_G = 2.4 \times 10^{18} \text{GeV} = 1$ is assumed.

The lowest component of the chiral superfield ϕ with the abuse of notation consists of two real scalar fields as $\phi = x + iy$. Note

$$\Omega_\phi = \frac{1}{2}(\Omega_x - i\Omega_y), \quad \Omega_{\phi\phi^*} = \frac{1}{4}(\Omega_{xx} + \Omega_{yy}), \quad (2)$$

where the subscripts denote differentiations with respect to them. The kinetic function is given by

$$K_{\phi\phi^*} = \frac{3}{\Omega^2}(|\Omega_\phi|^2 - \Omega_{\phi\phi^*}\Omega) \quad (3)$$

with $K_\phi = -3\Omega_\phi/\Omega$. The scalar potential in supergravity is given by

$$\begin{aligned} V &= e^K (K_{\phi\phi^*}^{-1} |W_\phi + K_\phi W|^2 - 3|W|^2) \\ &= -\left(\frac{3}{\Omega}\right)^3 \left(\frac{\Omega^2}{3(|\Omega_\phi|^2 - \Omega_{\phi\phi^*}\Omega)} \left| W_\phi - \frac{3\Omega_\phi}{\Omega} W \right|^2 - 3|W|^2 \right) \end{aligned} \quad (4)$$

for the kinetic function Eq.(3).

3 Large-field inflation

A tuning for large-field inflation can be given by making the kinetic term large [8], or the overall coupling small. Let us first consider a Lagrangian

$$\mathcal{L} = \frac{1}{2} A^2 \partial_\mu \tilde{\varphi} \partial^\mu \tilde{\varphi} - \tilde{V}(\tilde{\varphi}) \quad (5)$$

of a real scalar field $\tilde{\varphi}$ with a positive constant A . For the canonical field⁷ $\varphi = A\tilde{\varphi}$,

$$\mathcal{L} = \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - V(\varphi), \quad (6)$$

where $V(\varphi) = \tilde{V}(\varphi/A)$. The corresponding slow-roll parameters

$$\epsilon = \frac{1}{2}\left(\frac{1}{V}\frac{\partial V}{\partial\varphi}\right)^2, \quad \eta = \frac{1}{V}\frac{\partial^2 V}{\partial\varphi^2} \quad (7)$$

are given by

$$\epsilon = \frac{1}{2A^2}\left(\frac{1}{\tilde{V}}\frac{\partial\tilde{V}}{\partial\tilde{\varphi}}\right)^2, \quad \eta = \frac{1}{A^2\tilde{V}}\frac{\partial^2\tilde{V}}{\partial\tilde{\varphi}^2}. \quad (8)$$

The sizes of these parameters are rendered to be small for a large value of A (and large field value φ).

A naive guess to apply the above setup to a supersymmetric case might be to adopt a Kähler potential $K = A^2|\tilde{\phi}|^2$. However, this does not work since the potential in supergravity contains the overall factor $\exp K$, in particular, which does not flatten out for large A .

A resolution we propose is, instead, to adopt a distorted Kähler potential like

$$K = 2A^2\tilde{x}^2 + \tilde{y}^2, \quad (9)$$

for example, for a complex field $\tilde{\phi} = \tilde{x} + i\tilde{y}$. Then the corresponding potential can be flat along the y direction for a large value of A , since $K = 2x^2 + (y/A)^2$ for an approximately canonical field $\phi = A\tilde{\phi} = x + iy$ with $K_{\phi\phi^*} = 1 + 1/(2A^2)$. We note that a pre-inflation [9] would not drive the field y to its origin since the mass of y is suppressed by A^{-1} in this setup, which might be advantageous for the initial condition of y inflation to be realized.

The above tuning method is utilized only as a guiding scheme in this paper for more general tuning of Kähler potentials. Let us classify the superpotential according to the exponent of the inflaton ϕ around the vacuum $\langle\phi\rangle = 0$ (modulo Kähler transformations).⁸ Note that the Kähler transformation to change the exponent is singular and inappropriate for the present purposes.

⁷ The possible variation of the canonical field φ is consequently larger than the reduced Planck scale for large A even if that of the original field $\tilde{\varphi}$ is restricted to be within the reduced Planck scale.

⁸ The choice of the origin $\phi = 0$ as the vacuum is just a convention.

3.1 The case with $W = c$

We omit to present the Kähler potential in terms of the original field variable $\tilde{\phi}$ but provide it in terms of the rescaled one ϕ from the start. Namely, making the coefficients of the y -dependent terms tiny as above, let us consider, for instance,

$$\Omega = -3 + 2\sqrt{3}x - ax^4 - by^n + \sqrt{3}x^3n(n-1)by^{n-2}f(y), \quad (10)$$

where a and b are positive constants and $n(\geq 4)$ is even. The kinetic function Eq.(3) is nearly one for small $|x|$ and b . The absence of the quadratic and cubic terms⁹ guarantees the vanishing cosmological constant in the vacuum $\langle\phi\rangle = 0$ [10], while the supersymmetry is inevitably broken so that the inflaton is a Polonyi field with the gravitino mass $m_{3/2} \simeq |c|$ in this case.

We can take a sufficiently large a and

$$f(y) = \frac{2}{9 + 3by^n} - \frac{n(n-1)by^{n-2}}{12 + (nby^{n-1})^2} \quad (11)$$

to stabilize the trajectory $x = 0$ with the latter choice eliminating the y -dependent tadpole of x . This is not a genuine tuning to induce inflation but it simplifies the analysis considerably. For $x = 0$,

$$\Omega = -3 - by^n, \quad \Omega_\phi = \sqrt{3} + \frac{i}{2}nby^{n-1}, \quad \Omega_{\phi\phi^*} = -\frac{1}{4}n(n-1)by^{n-2}, \quad (12)$$

and due to small b , for up to moderately large $|y|$,

$$V = \frac{81|c|^2}{\Omega^2} \frac{\Omega_{\phi\phi^\dagger}}{\Omega_{\phi\phi^\dagger}\Omega - |\Omega_\phi|^2} \simeq \frac{3}{4}|c|^2n(n-1)by^{n-2}, \quad (13)$$

which turns out to be suitable for chaotic inflation, as is intended.

As for the case of the primordial inflation, the COBE normalization [1] implies

$$\left| \frac{V(\varphi_{N_0})^{\frac{3}{2}}}{V'(\varphi_{N_0})} \right| = 5.3 \times 10^{-4} \quad (14)$$

for a canonical field $\varphi = \sqrt{2}y$, where φ_{N_0} is the inflaton field value at the exit of the present horizon. For $n = 4$, we obtain a quadratic potential with $b\varphi_{N_0}^2|c|^2 = 7.5 \times 10^{-7}$, which implies large gravitino mass $m_{3/2} \simeq |c| \gtrsim 10^{15}\text{GeV}$.

⁹The absence of the y -linear term is a convention.

3.2 The case with $W = m\phi^2$

For instance, let us consider

$$\Omega = -3 + dx + \left(2 - \frac{d^2}{6}\right)x^2 - ax^4 + x^3 f(y). \quad (15)$$

The kinetic function Eq.(3) is nearly one for small $|x|$.

As is the case for the previous example, we can stabilize the trajectory $x = 0$ with the choice

$$f(y) = \frac{d^3}{54} + \frac{8d(2 - y^2)}{48 + 3d^2 y^2}. \quad (16)$$

For $x = 0$, we obtain

$$\Omega = -3, \quad \Omega_\phi = \frac{d}{2}, \quad \Omega_{\phi\phi^*} = 1 - \frac{d^2}{12}, \quad (17)$$

and the potential is given by

$$V = |m|^2 \left(4y^2 + \left\{ \left(\frac{d}{2} \right)^2 - 3 \right\} y^4 \right). \quad (18)$$

Under a fine-tuning¹⁰ $d = 2\sqrt{3}$ for $2 - d^2/6 = (d/2)^2 - 3 = 0$, this potential is quadratic and the COBE normalization Eq.(14) yields $\varphi_{N_0}^4 |m|^2 = 5.6 \times 10^{-7}$, which implies $|m| \sim 10^{13} \text{GeV}$.

4 Small-field inflation

Small-field models require rather different tuning to achieve a flat inflaton potential, which results in inflation with a wider range of physical parameter scales compared to the previous large-field case. In the following, we try to tune the potential of the inflaton φ so that it has a ‘saddle point’ at $\varphi = \varphi_*$ with $|\varphi_*| \ll 1$, where the slow-roll parameters satisfy $\epsilon(\varphi_*) = \eta(\varphi_*) = 0$ [3]. This is a sufficient condition for slow-roll inflation to be possible. Generally speaking, it is unnecessary fine tuning and merely provides an

¹⁰ For a fixed initial value of the inflaton field (determined by b -dependent terms), the total e -fold is larger in the case of quadratic potential than that in the case of quartic one. This is a candidate reason for such a secondary tuning in addition to the primary one of large A , or small $|b|$, to induce inflation. See also Ref.[4].

example of small-field inflation models that are realized by certain tunings. The inflaton potential around the ‘saddle point’ is given by

$$V(\varphi) \simeq V(\varphi_*) + \frac{1}{3!} V'''(\varphi_*) (\varphi - \varphi_*)^3, \quad (19)$$

where we assume $\varphi_* > 0$ and $V'''(\varphi_*) > 0$ without loss of generality.¹¹

The inflationary regime ends when the slow-roll condition ($\epsilon < 1$ and $|\eta| < 1$) is violated and such a point $\varphi = \varphi_f$ is given by

$$\varphi_f \sim \varphi_* - \frac{V(\varphi_*)}{V'''(\varphi_*)}. \quad (20)$$

The e -fold number N_e corresponding to the value φ_{N_e} of the inflaton field is given by

$$N_e \simeq \int_{\varphi_f}^{\varphi_{N_e}} \frac{V(\varphi)}{V'(\varphi)} d\varphi \simeq \frac{2V(\varphi_0)}{V'''(\varphi_0)} \frac{1}{\varphi_* - \varphi_{N_e}}, \quad (21)$$

which leads to

$$\varphi_{N_e} \simeq \varphi_* - \frac{2V(\phi_0)}{N_e V'''(\phi_0)}. \quad (22)$$

The amplitude of density fluctuations is given by

$$\left| \frac{V(\varphi_{N_0})^{\frac{3}{2}}}{V'(\varphi_{N_0})} \right| \simeq \frac{2V(\varphi_0)^{\frac{3}{2}}}{V'''(\varphi_{N_0})(\varphi_{N_0} - \varphi_*)^2}, \quad (23)$$

where N_0 denotes the e -fold of the present horizon. The COBE normalization Eq.(14) thus implies

$$\frac{V'''(\varphi_*)^2}{V(\varphi_*)} \simeq 1.1 \times 10^{-6} N_0^{-4}. \quad (24)$$

4.1 The case with $W = c$

Let us consider, for an illustration,

$$\Omega = -3 + 2\sqrt{3}x - ay^4 + by^5 - 2dy^6, \quad (25)$$

where a, b, d are positive constants. The kinetic function Eq.(3) is nearly one for small $|x|$ and $|y|$. The absence of the quadratic and cubic terms implies that the origin $\phi = 0$

¹¹ The initial condition for such inflation might be realized through primary inflations [1, 11].

yields a supersymmetry-breaking minimum with the vanishing cosmological constant [10], as does in the previous section.

For $x = 0$,¹² the scalar potential is given by

$$V = \frac{81|c|^2}{\Omega^2} \frac{\Omega_{\phi\phi^\dagger}}{\Omega_{\phi\phi^\dagger}\Omega - |\Omega_\phi|^2} \simeq 3|c|^2 \frac{3ay^2 - 5by^3 + 15dy^4}{1 - (3ay^2 - 5by^3 + 15dy^4)}. \quad (26)$$

For $b \ll d$, this potential possesses a ‘saddle point’ at

$$y_* = \frac{b}{8d} \ll 1 \quad (27)$$

when the following condition is satisfied:

$$a = \frac{5b^2}{32d}. \quad (28)$$

Then the potential around the ‘saddle point’ $\varphi_* = \sqrt{2}y_*$ is given by

$$V \simeq \frac{15b}{16\sqrt{2}}|c|^2\varphi_*^3 + \frac{15b}{4\sqrt{2}}|c|^2(\varphi - \varphi_*)^3. \quad (29)$$

By means of Eq.(24), the density fluctuations determine the overall scale as

$$|c|^2 \simeq 2.9 \times 10^{-9} N_0^{-4} b^{-1} \varphi_*^3. \quad (30)$$

Thus the gravitino mass is given by

$$m_{3/2} \simeq |c| \simeq 5.4 \times 10^{-5} N_0^{-2} \sqrt{b^{-1}\varphi_*^3} \sim 5 \times 10^{11} \text{GeV} \sqrt{b^{-1}\varphi_*^3}, \quad (31)$$

which can be small for small-field inflation. For example, $\varphi_* \sim 10^{-3}$ and $m_{3/2} \simeq |c| \sim 10\text{TeV}$ are realized by tuning $b \sim 10^6$ and $d \sim 10^8$, which is consistent to the framework of low-energy effective theory with a cutoff scale larger than φ_* .

Finally, we comment on cosmological problems. In this model, the inflaton mass is given by $m_\varphi \simeq 3\sqrt{a}m_{3/2}$. The parameter choice $a \lesssim 1$ causes the Polonyi problem for the gravitino mass $\lesssim 10\text{TeV}$ when the inflaton decay is Planck suppressed. For $a > 4/9$, gravitinos tend to be overproduced through the inflaton decay even if the gravitino is as heavy as of order 100TeV . Thus we are led to consider superheavy or superlight gravitino unless entropy is produced to dilute the primordial gravitino or the inflaton decay is enhanced, which may be also advantageous for baryogenesis at higher reheating temperature.

¹²As is the case in the previous section, additional higher-dimensional terms in Ω are required to stabilize this trajectory, which we do not show explicitly any more in this section.

4.2 The case with $W = m\phi^2$

Here we consider, for an illustration,¹³

$$\Omega = -3 + 2xyf + 2x^2 \left(1 - \frac{1}{3}y^2f^2\right); \quad f = a - by^2 + dy^4, \quad (32)$$

where a, b, d are positive constants. The kinetic function Eq.(3) is nearly one for small $|x|$ and $|y|$.

For $x = 0$, the scalar potential is given by

$$V = |m|^2 \{4y^2 - 3y^4 + (y^3f)^2\}. \quad (33)$$

For large b with $b \ll d$, this potential possesses an approximate ‘saddle point’ at

$$y_*^2 = \frac{5b}{14d} \ll 1 \quad (34)$$

when the following condition is satisfied:

$$a = \frac{25b^2}{84d}. \quad (35)$$

Then the potential around the ‘saddle point’ $\varphi_* = \sqrt{2}y_*$ is given by

$$V \simeq \frac{b^2}{6}|m|^2\varphi_*^{10} + \frac{5b^2}{126}|m|^2\varphi_*^7(\varphi - \varphi_*)^3. \quad (36)$$

By means of Eq.(24), the density fluctuations determine the overall scale as

$$|m|^2 \simeq 3.2 \times 10^{-6} N_0^{-4} b^{-2} \varphi_*^{-4}, \quad (37)$$

that is,

$$|m| \simeq 1.8 \times 10^{-3} N_0^{-2} b^{-1} \varphi_*^{-2} \sim 10^{12} \text{GeV} b^{-1} \varphi_*^{-2}. \quad (38)$$

For example, $b \sim 10^7$ and $d \sim 10^9$ result in $a \sim 10^5$, $\varphi_* \sim 10^{-1}$, and $|m| \sim 10^7 \text{GeV}$.

¹³ This respects a parity $\phi \rightarrow -\phi$, though such a symmetry is unnecessary just as an illustrative model for small-field inflation.

5 Conclusion

If supersymmetry and inflation are indeed operative in nature, supersymmetric inflation is expected to be a key ingredient to realize our present universe. Then knowledge of the variety of supersymmetric inflation models serves as a basis for characterizing the unique model among them chosen by nature.

We have categorized supersymmetric inflation models of chiral superfields according to the number of field multiplets and the exponent of the fields around the vacuum in the superpotential. In both the cases of large and small field inflations, we have also provided concrete examples of tuning schemes to realize flat potentials for slow-roll inflation in the case of a single chiral superfield. That is, tuned Kähler potentials are given, which result in flat inflaton potentials in each superpotential so categorized.

The examples of the tuning schemes we consider are rather different in two cases of large and small field inflations, where the variation of inflaton field is large and small, respectively, during inflation. In the large-field case, distorted Kähler potentials realize chaotic inflation by means of the monomial superpotentials with the exponents zero and two. In the small-field case, ‘saddle point’ inflation is realized by tuning Kähler potentials under the monomial superpotentials also with the exponents zero and two.

Although these tuning methods just provide mere examples of flattening the inflaton potentials, they at least show that such tunings are indeed possible. In view of the subtlety of fine-tuning problem in inflation mentioned in the first footnote, it might be expected that even these examples might have possible positions to describe nature.

Of course, further consideration is necessary in order to go beyond these somewhat artificial setups and arrive at realistic particle-physics models of inflaton. In particular, extensions to multi-field cases (with supersymmetry breaking) should also be investigated to obtain global view on the variety of supersymmetric inflation models.

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